On the influence of fractal dimension on radiation efficiency and quality factor of self-resonant prefractal wire monopoles

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Abstract. The influence of fractal dimension on radiation efficiency and quality factor of small self-resonant prefractal wire monopoles is investigated through simulations and measurements using the Wheeler cap method. Comparison of parameters for the same electrical size between prefractal and Euclidean antennas is also shown.

1. Introduction: small antennas and fractal dimension

Almost a decade ago, an intense research was engaged the improvement of antenna properties using fractal structures. The self-scalability and space-filling properties of some fractal curves opened the design of multifrequency antennas [1] and the development of efficient miniature antennas, respectively. Both undoubtful properties and promising characteristics are suitable (and very appealing) for the base stations and terminals of personal communications systems.

An antenna is said small when its size is made much smaller than its operating wavelength, in fact, when it can be enclosed into a radiansphere [2] (a sphere with radius \( a = \lambda / 2\pi \)). When working with small antennas, special attention deserve radiation efficiency (\( \eta \)), quality factor (\( Q \)) and electric size. The radiation pattern of the antenna is not of such importance because it does not differ from any other small antenna. Small antennas are inefficient by nature: their radiation resistance is low, they are difficult to match, and display high \( Q \) values (that means very narrow bandwidth), always worst than the fundamental limitation established by Chu and reexamined by McLean [3]. Efficient and wide bandwidth small antennas are the targets of our research.

In [4] Koch fractal monopoles were used as a first example of how fractals could be used to miniaturize an antenna reaching better performance, in terms of \( Q \), than the Euclidean straight monopole. More intricate dipole structures have been analyzed at [5] assessing the same conclusions. These results agreed with the hypothesis that the bandwidth of an antenna can be improved when the antenna efficiently uses the available volume of the virtual sphere that surrounds it ([6],[7]). In addition at [5] was concluded that “there exists a relation between the fractal geometric properties and the electromagnetic behavior of the antenna and that such properties can be readily employed to design useful antennas that might improve some features of common Euclidean ones”. The purpose of this work is to perform a systematic assessment of the characteristics of self-resonant prefractal wire antennas analyzing the influence of fractal dimension on \( \eta \) and \( Q \).

Self-resonant antennas do not require external compensation at their input terminals to balance their reactive energy. Fractal dimension is considered here in a loose sense, defined as the measure of the space-filling ability of the fractal antenna in terms of the self-similarity dimension [8]. Because of its degree of intricacy, the
ideal fractal cannot be built, and even in nature does not exist an ideal fractal. Our research is limited to **prefractals** with a low degree of iteration. Fortunately, this truncated complexity is unresolvable for the electromagnetic waves. Prefractals are considered to have the same fractal dimension that the limiting fractal.

### 2. Antenna description and results from simulations

To assess the relation between fractal dimension $D$ and $\eta$ and $Q$ several prefractal monopoles with different fractal dimension were fabricated. The study has been carried out using wire monopoles whose geometry can be described into a planar surface. These antennas provide a simple mean to fabricate (using conventional printed circuit technology) and analyze the behavior of fractals.

Prefractal geometries are constructed using thin wires and Iterative Function System algorithms (mathematically defined as an iterative set of affine transformations). The software used to model the prefractals was NEC, a moment-method code [9], suitable to solve the currents generated by thin wires and perfectly conducting surfaces.

Geometries investigated in this work range from $D=1$ to $D=2$. Simulated prefractals are Koch monopoles ($D=1.26$), Sierpinski Arrowhead monopoles ($D=1.58$), Hilbert monopoles ($D=2$) and Peano monopoles ($D=2$). The Sierpinski Arrowhead is a continuous fractal curve that generates the Sierpinski gasket. In this research, its input terminal is one of the ends of the curve. The Peano curve analyzed in this work, though strictly not a prefractal, is a variation of an standard Peano curve altered not to exist contact among any of its segments and to reduce even more its resonant frequency. All of the monopoles are made resonant at 800 MHz and simulated with copper wires of 0.2 mm-radius. Figure 1 shows the simulated performance at resonance of the antennas. They are gathered by families with dotted lines (radiation efficiency at left, and quality factor at right). Results reveal the reduction in size of the structures with the increasing iteration and fractal dimension, but antennas become less efficient and with reduced fractional bandwidth (higher $Q$). Both results are expected due to the increased ohmic resistance of the antennas (the length of the conducting wire increases with iteration and fractal dimension) and the reduced radiation resistance (the electric height of the antennas is reduced with iteration and fractal dimension).

These prefractal antennas are compared with a Euclidean straight monopole and some meander line loaded monopoles resonant at 800 MHz. All are designed with copper wire of 0.2 mm-radius. Simulations show how some intuitively generated (non-optimum) non-fractal monopoles achieve the same electric size but with better performance in terms of $\eta$ and $Q$ than prefractals do. Much better results should have been reached if any optimization technique had been used in the design.

### 3. Manufactured antennas and technique for $\eta$ and $Q$ measurement

The antennas were fabricated using conventional techniques for the manufacturing of printed circuits for electronics boards. A slim (0.2 mm) fiberglass substrate (FR4) was used as a support for the strips that configure the monopole antennas.
The strips are 0.35 mm wide and their thickness is 35 µm. Monopoles are welded on an 80x80 cm² ground plane and fedded with an SMA connector. Figure 2 shows the fabricated structures.

To measure $\eta$ and $Q$, the Wheeler cap method was used among others ([10]) because of its accuracy, repeatability and its quick measurement procedure. With a properly designed cap, and once measured the input impedances of the antenna with the shield on ($R_{in,1}$) and off ($R_{in,2}$) and using a network analyzer, the radiation efficiency and the quality factor are determined. At resonance $\omega_0$, the quality factor of the antenna can be obtained with equation (1)

$$Q = \frac{\omega}{2(R_{in,2} - R_{in,1})} \left[ \frac{dX_{in,2}}{d\omega} + \left. \frac{X_{in,2}}{\omega} \right|_{\omega_0} \right] = \frac{\omega}{2(R_{in,2} - R_{in,1})} \left[ \frac{dX_{in,2}}{d\omega} \right]_{\omega_0}$$  \hspace{1cm} (1)

where $X_{in,2}$ is the input reactance measured without the cap. This value can be compared with the ideal lossless value determined from the fundamental limit. Other methods of $\eta$ and $Q$ measurement can be used to resolve doubtful results.

Preliminary results from the use of an small cylindrical cap agree with simulations, showing a loss of efficiency and an increase in $Q$ when the iteration number and the fractal dimension of the self-resonant prefractal wire monopoles increase. Better performance from the Euclidean structures was observed. Accurate results with a wider cap will be shown in the presentation.

4. Conclusions

Increasing the fractal dimension of a self-resonant prefractal wire monopole does not improve its performance against conventional Euclidean antennas in terms of radiation efficiency and quality factor. In addition, the use of fractal structures limits the degrees of freedom in the design of the monopoles due to the fixed relation between resonant wavelength and geometric dimensions.

Acknowledgements

This work has been partially supported by the Spanish Government through grant TIC 2001-2364-C03-01, the Ministerio de Ciencia y Tecnología through the Ramon y Cajal Program (2002), the European Community through the project IST-2001-33055, and the Departament d’Universitats, Recerca i Societat de la Informació (DURSI) de la Generalitat de Catalunya.

References


Figure 1. Simulated $\eta$ and $Q$ of self-resonant prefractal wire monopoles of different fractal dimension compared with an Euclidean structures. Dotted lines join values at resonance of antennas from the same family.

Figure 2. Fabricated structures: (a) Koch monopoles; (b) Sierpinski Arrowhead monopoles; (c) Hilbert monopoles; (d) Peano variant 2 monopoles; (e) Euclidean $\lambda/4$ monopole, and (f) Meander Line Loaded Monopoles.