

Numerical analysis of composite finite arrays of Split Ring Resonators and thin strips

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Abstract: Composite finite perfectly conducting (PeC) arrays of thin strips and Split Ring Resonators (SRR) with a Method of Moments integral formulation and triangular patches [1] have been analyzed numerically. This paper presents the effect on the accuracy of the solution due to the type of meshing of the SRR's, the stopping residual error of the iterative algorithm GMRES and the accuracy of the PeC-EFIE operator.

INTRODUCTION

The composite arrays of thin strips and Split Ring Resonators (SRR) can be used to reproduce properties inherent to specific dielectric media. It is particularly interesting the construction of left-handed metamaterials, which, at the resonant frequency, show negative effective dielectric constants, with no known physical correspondence [2][3].

Since these structures have quite high electrical dimensions, a high number of unknowns is required. Furthermore, since the SRR's are electrically small elements inside a resonant structure, a very fine discretization may be required to expand the current correctly. Therefore, the matrix becomes ill-conditioned. Accurate currents and radiation patterns are indispensable for the sake of the reliability of the later computation of the corresponding effective dielectric constants

RESULTS

The two composite finite structures analysed in this paper are the *2-layer array* (see Fig. 1) and the *1-layer array* (readily derived from the previous one by removing the second combined array in the $-x$ direction). Two different meshings of the SRR's [3] have been used (see Fig. 2) for each structure. The number of triangles required to mesh the *1-layer array* and the *2-layer array* become, respectively, 16512 and 47136 with the discretization (a) and 21696 and 48576 with the discretization (b). The thin strips are meshed with the same number of triangles for both arrays.

A block-ILU [4] preconditioning scheme has been used to speed up the iterative inverting algorithm adopted (GMRES). For the *2-layer array* and the *1-layer array*, the impedance elements between basis functions at distances bigger than 35 mm and 30 mm, respectively, have been dropped to build the preconditioner. The incomplete LU transform has been carried out with 40 blocks and 32 blocks, respectively, and a dropping-tolerance of $5e-4$. The Multilevel Fast Multipole Method [5] has been applied to accelerate the matrix-vector product at each iterative step. The frequency of study is $F=5.4$ GHz.

(a) PeC-EFIE operator

Two versions of the PeC-EFIE operator have been programmed. The *numerical operator* uses a 4-point gaussian quadrature rule to integrate in source triangles. The *analytical operator* carries out the source integration analytically for the $1/R$ term [6]

and uses 1 point for the low-order remaining part. Also, a 4-point field-gaussian quadrature rule is used in the matrix elements corresponding to field triangles closer than 9 mm from the source triangles. The block-ILU preconditioning scheme is developed from the impedance elements yielded by the *numerical operator*. In Table I, the number of iterations required to converge to a relative residual error of 1% is shown for each operator and for the geometries *1-layer array-(b)*, *2-layer array-(a)* and *2-layer array-(b)*.

	<i>1-layer (b)</i>	<i>2-layer (a)</i>	<i>2-layer (b)</i>
<i>Numerical</i>	74	152	No convergence
<i>Analytical</i>	79	177	169

Table I – number of iterations to converge to 1% of relative residual error

(b) Expansion of the current

For the *1-layer array* with a relative residual error of convergence of 1% and the *numerical operator*, the radiation patterns for both discretizations (a) and (b) are very similar (see Fig. 3). The directivities for the cases *1-layer array-(a)* and *1-layer array-(b)* become 4.03 dB and 3.78 dB, respectively.

It is interesting to remark how for the *2-layer array* with a relative residual error of 1% and the *analytical operator*, the radiation patterns for both discretizations (a) and (b) become remarkably different (see Fig. 4). Indeed, in these cases, the directivities become 3.07 dB and 4.84 dB, respectively.

(c) Residual error criterion

In Table II, the directivities of the radiation patterns for the cases *1-layer array-(b)* and *2-layer array-(b)* (see Fig. 5) for relative residuals errors of 1% and 0.1 % are shown.

	<i>1-layer array-(b)</i>	<i>2-layer array-(b)</i>
Residual error = 1%	3.78 dB	3.07 dB
Residual error = 0.1%	3.86 dB	3.23 dB

Table II – Directivities of the radiation patterns and the discretization (b) for the residual errors 1% and 0.1 %

The relative errors of the current with residual 1% respect to the current at 0.1% are for the cases *1-layer-(b)* and *2-layer array-(b)* 12.37% and 15.22%, respectively.

CONCLUSIONS

A version of the PeC-EFIE operator with analytical source integration is useful to speed-up the convergence for the very finely meshed case *2-layer array-(b)*. The *2-layer array* appears to be very sensitive to the expansion of the current as shown by the fact that both discretizations (a) and (b) provide very different results for the same residual error (1%). Although the radiation patterns for *2-layer array-(b)* and residual errors of 1% and 0.1% barely vary over the xy and xz cuts, an even more restrictive criterion for the residual error must be carried out because the difference between the directivities of the patterns and the errors of current are not small enough yet. We are currently investigating if the observed discrepancy in the *2-layer array* results is due to the discretization or to the chosen convergence criterion.

ACKNOWLEDGEMENTS

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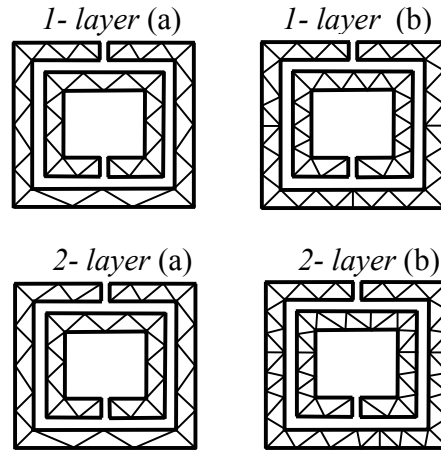


Figure 1.- Discretizations (a) and (b) of the SRR's for the 1-layer and the 2-layer arrays

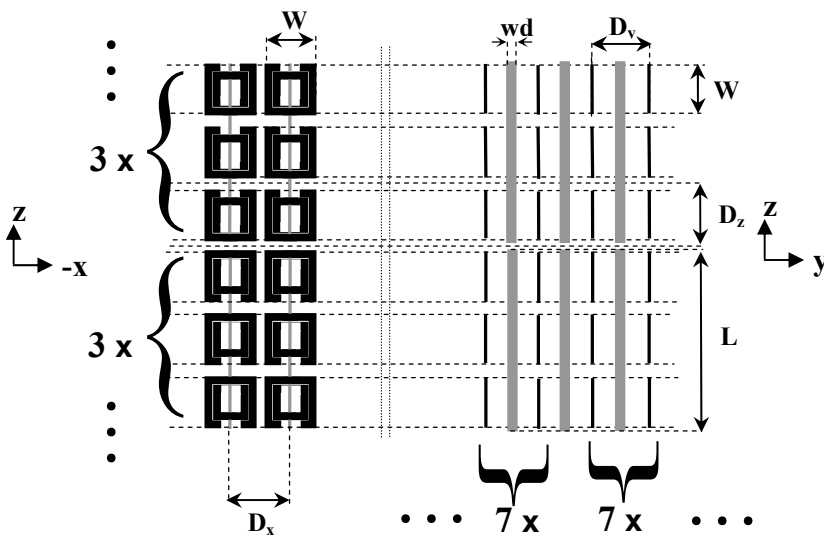


Figure 2.- XZ and YZ sections of the array of SRR and thin strips.
 $D_x=8\text{mm}$, $D_y=8\text{mm}$, $D_z=10\text{mm}$, $W=6.6\text{mm}$, $L=27.52\text{mm}$, $wd=1.5\text{mm}$

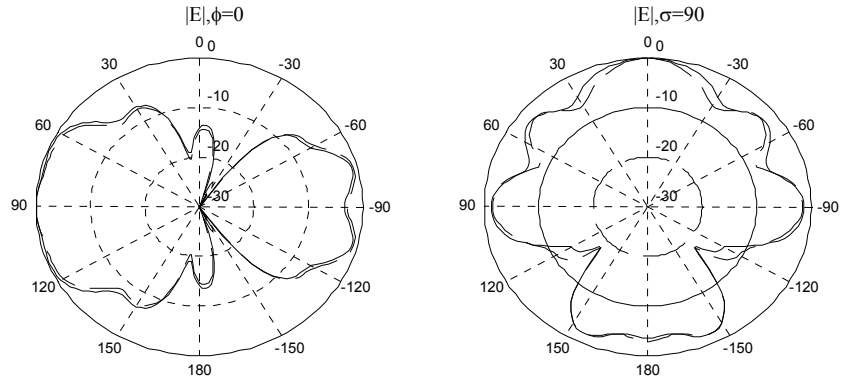


Figure 3.- Radiation pattern of the for the *1-layer array* with the meshing (a) (—) and the meshing (b) (--) with an elementary ideal dipole z-oriented at 1λ of distance in the +x-direction

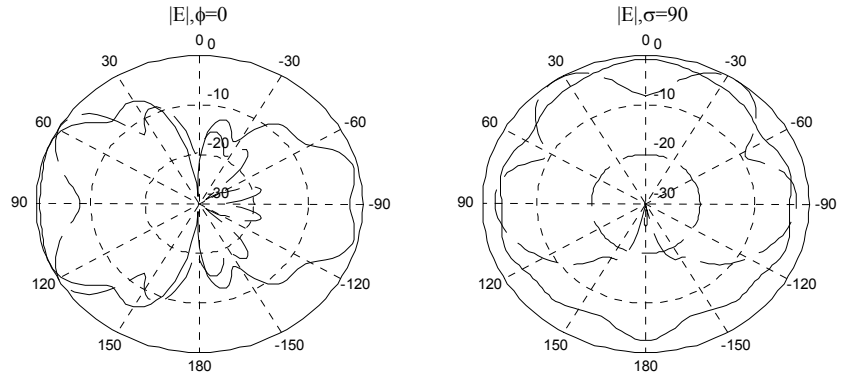


Figure 4.- Radiation pattern of the for the *2-layer array* with the meshing (a) (—) and the meshing (b) (--) with an elementary ideal dipole z-oriented at 1λ of distance in the +x-direction

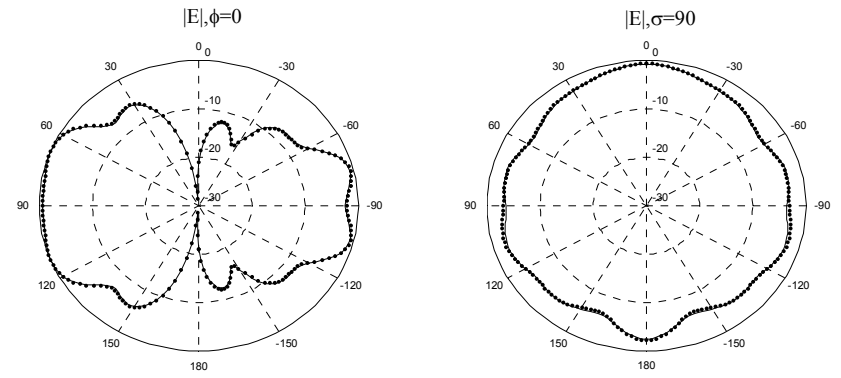


Figure 5.- Radiation pattern of the for the *2-layer array* and the meshing (b) with residual error of 1% (•••) and 0.1% (—) with an elementary ideal dipole z-oriented at 1λ of distance in the +x-direction