

Efficient Full-Kernel Evaluation in the Thin-Wire Electric Field Integral Equation

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Abstract: New expressions for the numerical evaluation of the ‘full’ or ‘exact’ thin wire kernel are presented. They allow for fast and accurate Method of Moments calculations of wire models with arbitrary segment length/radius ratios. The new expressions are shown to converge with respect to mesh refinement for the case of a slab of metamaterial.

INTRODUCTION

Antennas and scatterers that are made of long and electrically thin perfectly conducting structures are commonly analyzed with the thin wire formulation of the Method of Moments (MoM). This involves a meshing of the structure into straight cylindrical wire segments and computing the mutual impedances of each of them. A critical parameter for the accurate computation of the mutual impedances is the ratio of the segment length Δ and wire radius a . For $\Delta/a \gg 1$, a simple and fast 1-D approximation can be used, the ‘reduced’ kernel approximation [1], as opposed to the ‘full’ or ‘exact’ 2-D kernel. Well known electromagnetic simulation codes such as NEC and WIPL [2] use approximations that work well for $\Delta/a \gtrsim 1$, but fail for smaller ratios. Exact series solutions have been published, that are valid for any value of Δ/a , but these are generally cumbersome to evaluate [3]. This paper presents an approximation that is highly accurate for any value of Δ/a and only about twice slower than the 1-D approximation.

NEW EXPRESSIONS FOR THE EXACT KERNEL

In order to compute the mutual impedance between two wire segments, one must evaluate the scalar and vector potentials due to the current on one segment, at the location of the other. We adopt the usual assumptions of the thin wire model: surface current densities that are 1) Constant along the wire circumference, and 2) Always parallel to the segment axis. The axial dependence of the current is assumed to be linear. Then, as shown for example in [4], the difficult part of evaluating the potentials can be reduced to computation of the following two integrals (for a segment aligned with the z -axis):

$$F_{\phi}(r,z) = \int_0^L \int_0^{2\pi} 1/R \, dz d\phi \quad (1)$$

and

$$F_A(r,z) = \int_0^L \int_0^{2\pi} z/R \, dz d\phi \quad (2)$$

with

$$R = \sqrt{(z^2 + r^2 + L^2 - 2r \cos(\phi))}, \quad (3)$$

where (1) and (2) must be evaluated for $z = z_1$ and for $z = z_2$, the segment end-points. The entire geometry is implicitly scaled to $a=L$. We have found the following expressions for (1) and (2):

$$\begin{aligned} F_{\phi}(r,z) \approx & 2k_0/\pi \{ a_1 k + a_2 k k_0^2 + a_3 k k_0^4 + a_4 k^3 + a_5 k_0^2 k^3 + a_6 k^5 \\ & + (a_7 + a_8 k_0^2 + a_9 k_0^4) \sqrt{(1 - k_0^2)} \arctan(k/\sqrt{(1 - k_0^2)}) \\ & + a_{10} \operatorname{arctanh}(k/k_0)/k_0 \\ & + (a_{11} k + a_{12} k k_0^2 + a_{13} k k_0^4 + a_{14} k^3 + a_{15} k_0^2 k^3 + a_{16} k^5) \ln(1 - k_1^2) \} \end{aligned} \quad (4)$$

and

$$F_A(r,z) \approx 4\sqrt{r}/(\pi k_1) \{ 1 + b_1(1 - k_1^2) + b_2(1 - k_1^2)^2 - (b_3(1 - k_1^2) + b_4(1 - k_1^2)^2) \ln(1 - k_1^2) \} \quad (5)$$

in which

$$k_0 = \sqrt{(4r/(r+L)^2)}, \quad k_1 = \sqrt{(4r/(z^2 + (r+L)^2))}, \quad k = \sqrt{(k_0^2 - k_1^2)}. \quad (6)$$

The coefficients a_n , b_n are given in appendix A. These two expressions are easy and fast to evaluate, since they only contain standard functions, and accurate: the relative error is below 10^{-4} for all r and $\Delta/a \geq 10^{-4}$.

RESULTS

We demonstrate the usefulness of the new expression with an analysis of a slab of metamaterial [5]. The slab under investigation is shown in Fig. 1. It consists of a lattice of x - z oriented Split Ring Resonators (SRRs) and straight wires. The SRR inner ring central axis has a radius of 2.1 mm, the outer a radius of 3.3 mm. The inner ring has a gap (0.3 mm) at the bottom, the outer at the top. The lattice periodicity is 8 mm in y - and z -direction. All the wires have a radius of 0.5 mm. The slab was illuminated with a z -oriented dipole, centered with respect to the slab at a distance of one wavelength in x -direction.

In order to determine which mesh parameters to use, we started by investigating the resonant frequency of a single SRR versus mesh size. As Fig. 2 shows, the region near the gaps needs to be meshed very finely. We obtained the same result for a uniform mesh of 8528 segments ($\Delta/a=0.0156$), and a mesh of 539 segments ($\Delta/a=0.25$ overall, but refined to $\Delta/a=0.001$ near the gaps). Using the latter meshing parameters for the entire slab (with $\Delta/a=0.25$ for the straight wires), we found it resonates at $f_{res} = 4.91$ GHz. This is different from the resonant frequency of a single SRR, because of the interaction between the elements.

In Fig. 3, the radiation pattern of the dipole plus slab at resonance is shown, together with the patterns when only the SRRs and only the straight wires are present. As predicted by the theory of metamaterials, the SRRs and the straight wires separately block the incoming field, while the metamaterial allows propagation in the forward direction.

CONCLUSION

Two new expressions are presented, that provide a fast and accurate evaluation of the 'exact' thin wire kernel for wire segments with radially constant surface current distribution and linear axial dependence. The expressions are valid for arbitrary values of Δ/a . An example of a Method of Moments analysis of a slab of metamaterial is given, showing convergence with respect to mesh refinement, and exhibiting the theoretically predicted behavior at the resonant frequency.

APPENDIX A

The coefficients a_n and b_n in Eqs. (4) and (5) are given by: $a_1 = 2.3945507$, $a_2 = -2.2986216$, $a_3 = 0.7203178$, $a_4 = 0.5511354$, $a_5 = -0.3938803$, $a_6 = 0.1152864$, $a_7 = -2.0024456$, $a_8 = 1.2328766$, $a_9 = -0.2302175$, $a_{10} = 1.5707964$, $a_{11} = -1.2950533$, $a_{12} = 1.0109890$, $a_{13} = -0.2158289$, $a_{14} = -0.3369963$, $a_{15} = 0.1438859$, $a_{16} = -0.0431658$, $b_1 = 0.4630151$, $b_2 = 0.1077812$, $b_3 = 0.2452727$, $b_4 = 0.0412496$. (The b_n coefficients are taken from [6], paragraph {17.3.35}).

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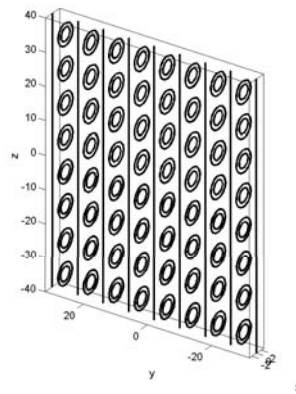


Fig. 1. Slab of metamaterial made of straight metal wires and SRRs.

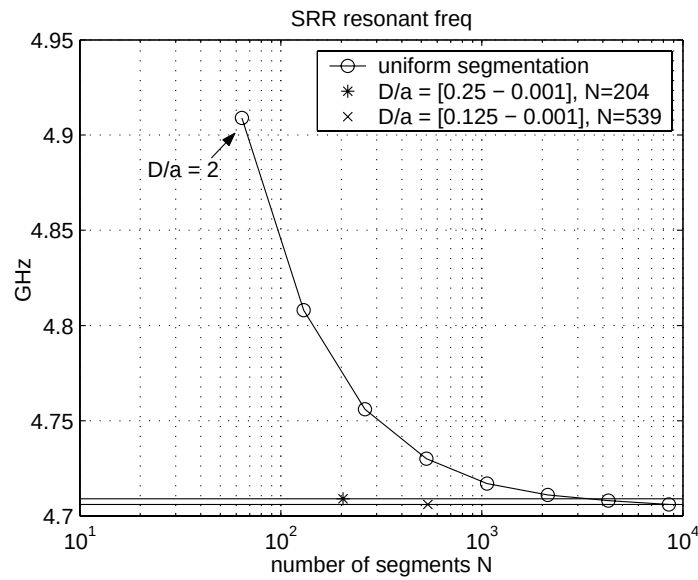


Fig. 2. Resonant frequency of SRR versus segment length for uniform segmentation. Also shown are results when a very fine mesh ($\Delta/a=0.001$) is used near the gaps.

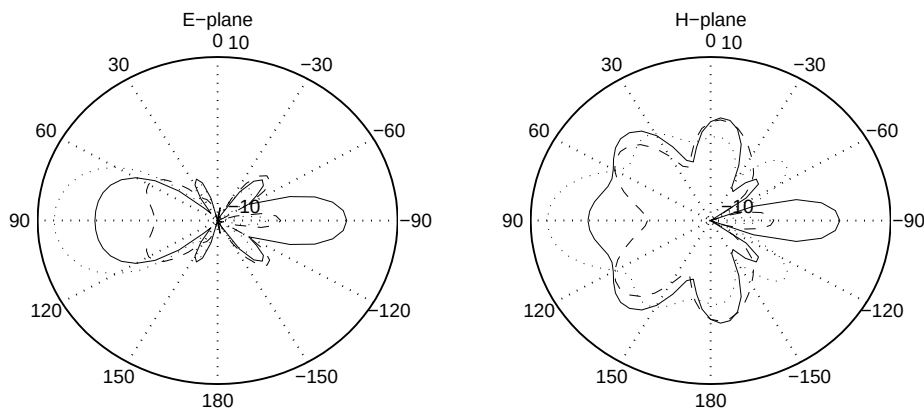


Fig. 3. Radiation pattern of straight wires (dashed line), SRRs (dotted line) and metamaterial (wires plus SRRs, solid line), illuminated at the resonant frequency. The forward direction is at -90 degrees.