

THE HILBERT CURVE AS A SMALL SELF-RESONANT MONOPOLE FROM A PRACTICAL POINT OF VIEW

J. M. González-Arbesú,^{1,2} S. Blanch,^{1,2} and J. Romeu^{1,2}

¹ Electromagnetics & Photonics Eng. Group
Universitat Politècnica de Catalunya
c/Gran Capità s/n, Mòdul D3, Campus Nord
08034 Barcelona, Spain

² Centre de Tecnologia de Telecomunicacions de Catalunya (CTTC)

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ABSTRACT: Prefractal Hilbert curves are analyzed in terms of radiation efficiency, quality factor, and electrical size for their use as small self-resonant monopole antennas. The structures show better performance as loads of monopoles than as radiating structures. © 2003

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Key words: monopole antennas; small antennas; wire antennas

1. INTRODUCTION

Hilbert prefractal curves applied to the design of antennas are currently receiving much attention [1–3]. The hypothesis that fractal curves can be used to design efficient small antennas [4], and that the space-filling properties of the Hilbert curve can be used to design optimum miniature antennas, have not been proved yet.

Investigations on the Hilbert curve as an antenna began using bidimensional curves in a dipole configuration and were aimed at the prediction of their resonant frequencies [5, 6] and higher-frequency compression factor [7, 1]. It was not until [8] that a three-dimensional (3D) Hilbert curve in a dipole configuration was introduced and its behavior analyzed in terms of radiation resistance η and quality factor Q at resonance. It was concluded from this work that, although a 3D Hilbert dipole has a small resonant frequency, it also has a high quality factor that is far from the fundamental limit.

Recent works [2, 3] compare bidimensional Hilbert fractal curves in dipole and monopole configurations, respectively, with other Euclidean structures to conclude that the prefractal antennas do not exhibit a resonant frequency lower than any other antenna of similar size; therefore, their self-similar and plane-filling nature will not be the key to more effective antenna designs.

In our work, several configurations of the Hilbert curve are analyzed and compared to other nonfractal antennas to assess its behavior. The behavior of the Hilbert monopole as a self-resonant small antenna is verified by simulations and measurements. The influence of the monopole orientation is also studied to reveal that the commonly analyzed configurations are not optimum.

Three-dimensional Hilbert antennas are also studied to assess their high losses and quality factors as monopole antennas. In this sense, a wide range of simulations are carried out with bidimensional Hilbert curve configurations in order to reveal their better performance as loads of monopoles. Comparisons with standard shorted monopoles are shown.

2. THE HILBERT MONOPOLE AS A SMALL PREFRACTAL ANTENNA

Hilbert curves are built through an iterative procedure (called networked iterative function systems, or networked IFSs) that generates almost self-similar structures [9]. In addition, Hilbert

curves are space-filling curves, meaning that in the limit the fractal curve fills the whole space. The capability to pack wires in a small space following a Hilbert curve is very appealing for manufacturing miniature antennas. But, although this capability is interesting as it reduces resonant frequencies, it is not enough. Other parameters have to be taken into account: radiation efficiency and quality factor.

Bidimensional prefractal Hilbert monopoles for the first four iterations (H-1, H-2, H-3 and H-4) are depicted at Figure 1, and their simulated radiation efficiency and quality factor at self-resonance (without any external loading) are shown in Figure 2. Radiation efficiency is reduced at each growth stage of the prefractal: the increase of the wire length with iteration also increases the ohmic resistance, and the smaller resonant frequency reduces the radiation resistance of the antenna. The intense coupling among the segments of the prefractal causes the high-quality factor values (see Table 1) to be far from the fundamental limit stated by Chu [10] for linear polarized antennas. Otherwise, the electric size at resonance of the antenna ($k_0 a$, where k_0 is the free-space wavenumber at resonance and a is the radius of the smallest sphere that encloses the monopole and its image on the ground plane) is highly reduced at each iteration, always in the small antenna region ($k_0 a < 1$). The performance of these Hilbert monopoles is compared with a conventional $\lambda/4$ monopole and other non-fractal intuitively generated monopoles (derived from meander-line monopoles) of the same height. It is shown that Hilbert antennas are not optimum designs and that the prefractal design constrains the electric sizes $k_0 a$ of the monopoles to a reduced set of values.

Simulations are carried out using NEC-2D, a method of moments (MoM) software [11] useful for analyzing wire structures. In our case, the structures are monopole configurations on a perfect and infinite ground plane. Monopoles were simulated with 0.2-mm-radius copper wires, and all of them were 89.80 mm in height. A 2-mm segment was provided at the base of the monopoles to simulate the welding segment for an SMA connector in a practical design.

3. COMMENTS ABOUT THE HILBERT MONOPOLE ORIENTATION

Previous references on the Hilbert monopole [1, 3] show different orientations for the same structure. Although the general trends of the Hilbert monopoles are the same irrespective their orientation, a closer look at their performance reveals some advantages for the structure analyzed in this work. These different configurations are depicted at Figure 3.

Figure 4 shows the relationship between the electric size $k_0 a$ of the antennas versus the electric length of wire at resonance $k_0 L$

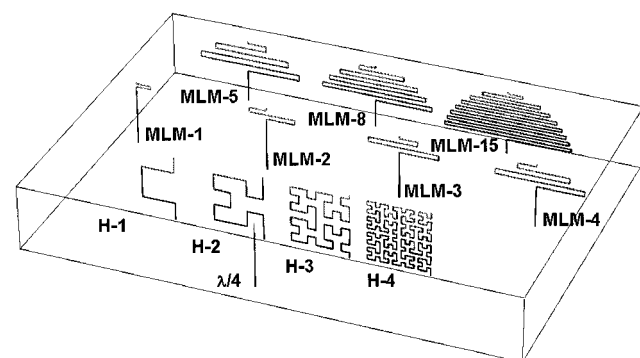


Figure 1 Hilbert monopoles compared with a conventional $\lambda/4$ monopole and other non-fractal and intuitively generated monopoles

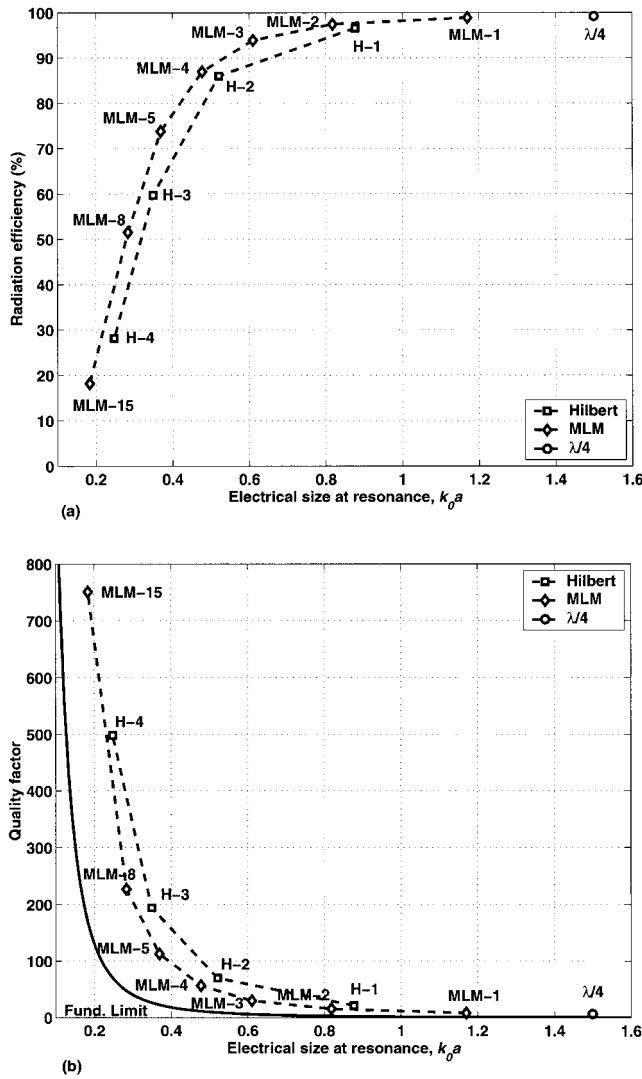


Figure 2 (a) Radiation efficiency and (b) quality factor of Hilbert monopoles at self-resonance compared with a $\lambda/4$ monopole and other non-fractal and intuitively-generated monopoles

needed to build the resonant Hilbert prefractal antennas. This figure shows that the horizontal orientation has a better miniaturization capability (that is, lower k_0a values for almost the same k_0L) than the rotated version. The vertical orientation is a com-

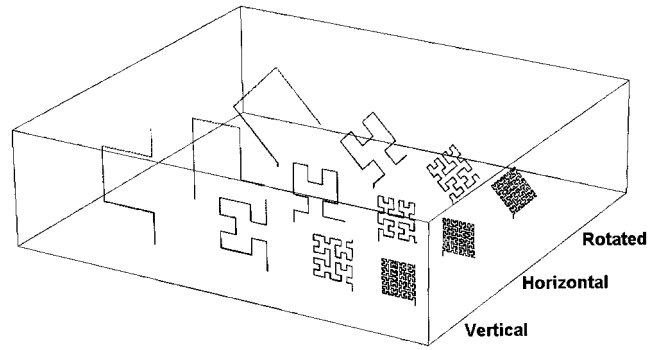


Figure 3 Hilbert monopoles compared according to their orientation. All of them are self-resonant at the same frequency

promise solution among them. The vertically oriented monopoles used in this work reveal a better behavior for prefractal orders greater than 2, as Figure 5 shows. The quality factor and efficiency are better for this orientation. Computed results are from 0.12-mm-radius copper wire monopoles resonant at 800 MHz.

4. THE HILBERT CURVE AS A 3D ANTENNA

The 3D Hilbert curve is an extended concept of the Hilbert curve generated from the continuous mapping of a segment into a cube [12]. Configurations analyzed in this section are monopoles whose structure is one half of the 3D Hilbert curve (as introduced in [8]). Their feeding point is at the lowermost segment (equivalent to the middle segment of the dipole structure) of the antennas. Simulated prefractals are depicted in Figure 6. Prefractal monopole efficiency and quality factor from the first to the third iteration of the Hilbert curve are summarized in Table 2 and compared with the fundamental limit values for a linearly polarized antenna with the same electrical size as the prefractal. As expected, the quick increase in wire length and consequently ohmic resistance, the reduction of the radiation resistance, and the coupling among segments of the monopole also make the 3D Hilbert monopole a poor and impractical antenna, despite the higher reduction achieved at its electrical size. Radiation efficiency is low (always under 80%) and quality factor is very high. All of the monopoles have the same overall height (89.8 mm) and have been simulated with 0.2-mm-radius copper wire.

Figure 7 shows the current distributions along the wire of the monopole for the first three iterations of the prefractal. These simulated current distributions on the wire at first resonance help

TABLE 1 Comparison of Prefractal Hilbert Wire Monopoles with Meander-Line Loaded Monopoles and $\lambda/4$ Monopoles at Self-Resonance (All Monopoles Have the Same Height)

Antenna	Total Wire Length (cm)	Electric Size at Resonance, ka	Resonant Frequency (MHz)	Radiation Resistance at Resonance (ohm)	Input Resistance at Resonance (ohm)
$\lambda/4$ Monopole	89.80	1.50	796.5	35.9	36.2
Hilbert-1	17.78	0.88	465.9	11.9	12.3
Hilbert-2	35.38	0.52	270.9	3.6	4.2
Hilbert-3	70.58	0.35	174.6	1.4	2.4
Hilbert-4	140.99	0.25	121.1	0.7	2.6
MLM-1	12.97	1.17	621.6	30.9	31.2
MLM-2	21.41	0.82	434.6	18.3	18.8
MLM-3	32.60	0.61	324.3	10.3	10.9
MLM-4	45.78	0.48	253.7	5.9	6.8
MLM-5	62.60	0.37	196.2	3.2	4.3
MLM-8	99.94	0.28	150.6	1.9	3.7
MLM-15	202.57	0.18	97.6	0.6	3.4

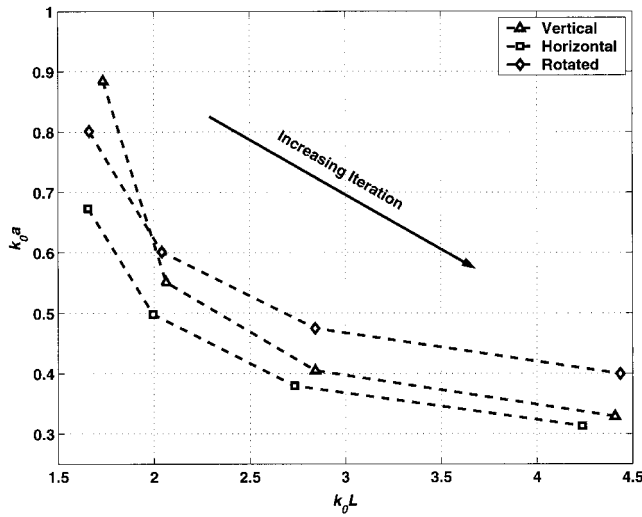


Figure 4 Compared electric size vs. electric length of Hilbert monopoles according to their orientation. All of them are self-resonant at 800 MHz

us to understand the reason for such a low efficiency. As the iteration increases, the current distribution on the wire changes from a sinusoidal distribution (as in a $\lambda/4$ monopole) to a more uniform current distribution in the first segments of the prefractal, in agreement with the results shown at [8]. The radiation mainly occurs in the first segment of the 3D Hilbert prefractal while the rest of the structure acts as a load. As the number of iterations is increased, the length of the radiating segment becomes smaller and the radiation resistance is reduced. Moreover, the wire length of the nonradiating part of the 3D Hilbert antenna is increased and also has higher currents. The result is an increase of ohmic losses. Altogether, these factors account for the reduction of efficiency as the number of iterations increases.

5. THE HILBERT CURVE AS LOADING OF MONOPOLES

Improvements in efficiency and quality factor observed when measuring Hilbert monopoles of high prefractal order at different heights of the ground plane suggested that Hilbert curves could be useful as top loads for shorting monopoles.

Monopoles loaded with Hilbert prefractals from iterations 1 to 3 are depicted in Figure 8 and simulated results for efficiency and quality factor are shown in Figure 9. Dashed lines in Figure 9 join values of efficiency and quality factor at self-resonance for the same prefractal (H-i) when the relative size of the prefractal load to the height of the monopole is changed. All the analyzed configurations have the same total height, 89.8 mm. The lowermost 2% of the monopole height is reserved to simulate the segment required for the welding of the feeding pin.

As observed in Figure 9, higher radiation efficiencies are obtained when small loads and long monopoles are used. Electrically smaller self-resonant monopoles are achieved by increasing the relative size of the Hilbert curve, but poorer values of radiation efficiency and quality factor are expected. Better values of quality factor are reached when increasing the iteration of the prefractal for almost the same radiation efficiency, however, the improvement is not significant.

The reduction in size achieved with the prefractal loading respective to the conventional $\lambda/4$ monopole ($k_0 a = 1.1$ in front of $k_0 a = 1.5$) is not negligible for almost the same efficiency and quality factor. At these size reductions, the same values are achieved with a banner monopole and conventional circular top-

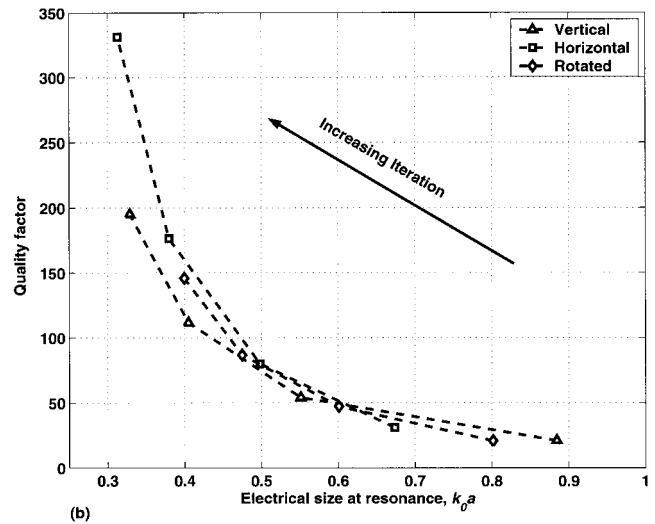
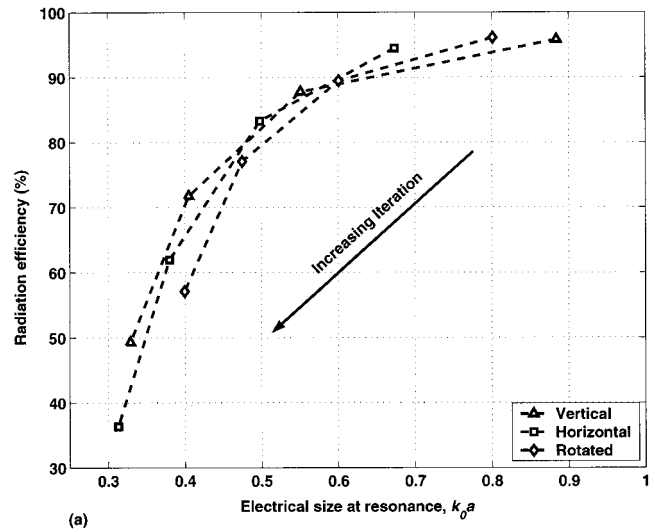


Figure 5 (a) Radiation efficiency and (b) quality factor of Hilbert monopoles at self-resonance compared according to their orientation. All of them are self-resonant at 800 MHz

loading plate, but the latter cannot be simply manufactured with PCB photo-etching technology. Figure 9 also shows that the conventional circular plate top-loaded monopole (TLM) only achieves a maximum reduction of $k_0 a$ closer to 0.4. At this point the loaded monopole has a very short feeding pin and a large plate, thus becoming more a patch than a small monopole. The banner mono-

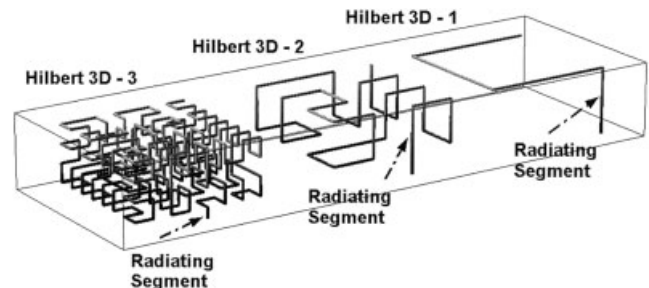


Figure 6 Three-dimensional Hilbert monopoles. Feeding is from the lower segment of each structure. The segment that mainly contributes to radiation is signaled

TABLE 2 Parameters at Self-Resonance of the 3D Prefractal Hilbert Wire Monopoles (All Have the Same Height)

Antenna	Total Wire Length (cm)	Electric Size at Resonance, ka	Resonant Frequency (MHz)	Radiation Resistance at Resonance (ohm)	Input Resistance at Resonance (ohm)	Efficiency (%)	Quality Factor	Fundamental Limit, Q
Hilbert 3D-1	61.78	0.71	127.5	2.47	3.24	76.6	126	4
Hilbert 3D-2	184.99	0.25	51.4	0.16	1.67	9.4	2091	70
Hilbert 3D-3	642.61	0.10	21.0	0.01	3.07	0.5	31966	1088

pole has a greater limitation in its size reduction, with k_0a approximately 0.8 the minimum size for this kind of structure.

When the results of Figures 2 and 9 are compared, the results show that for a given antenna size k_0a , better results are obtained when the Hilbert prefractal is used as a top-load than as an antenna. Although better results are always found with the meander-line-loaded monopoles (MLM).

6. CONCLUSION

Hilbert curves are space-filling curves thought to be helpful for designing small antennas. However, their shape provokes a strong cancellation of radiation, resulting in high Q values and a small radiation resistance. This small radiation resistance, along with the

large ohmic resistance (caused by the large wires needed to fabricate the structure) makes Hilbert antennas impractical designs for a radiating system.

Although the high quality factor of Hilbert curves makes them unappealing as radiators, their use as loads on top-loaded monopoles is suggested. Using Hilbert curves as loads of monopoles (rather than radiators by themselves) higher radiation efficiencies and quality factors are achieved, but at the expense of lower reductions of size than achievable as radiators.

Nevertheless, the use of Hilbert curves means no improvement against conventional designs, because better values of radiation efficiencies and quality factors for the same electrical sizes are

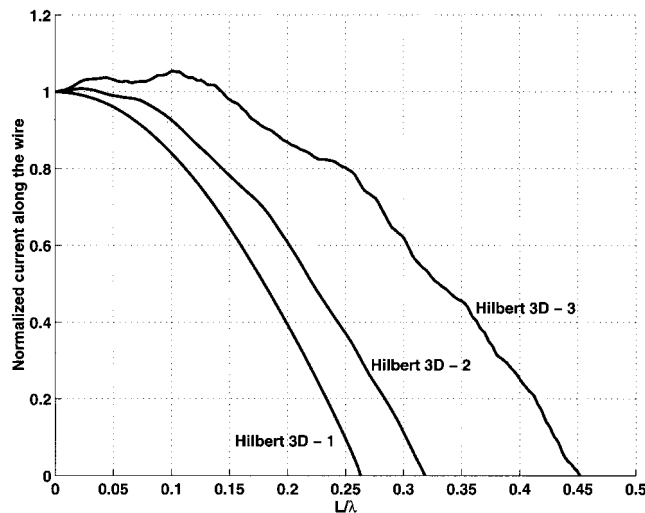


Figure 7 Current distributions along the wires that build the 3D Hilbert monopoles

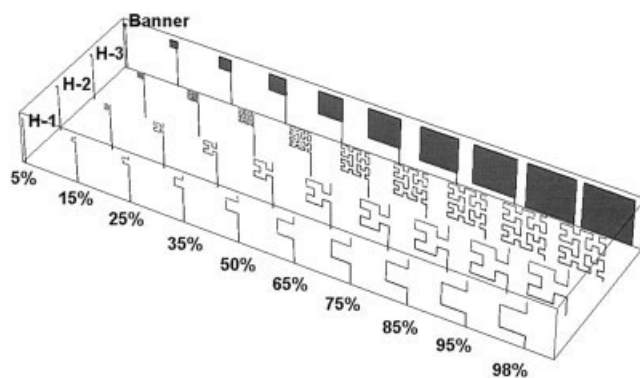


Figure 8 Hilbert curves as top loads of monopoles compared with banner monopoles. All of the structures have the same overall height. The percentage indicates the relative height of the monopole occupied by the prefractal and the banner, respectively

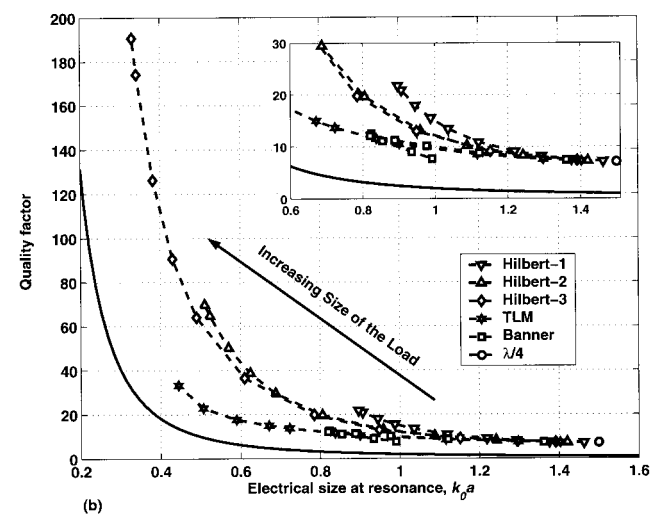
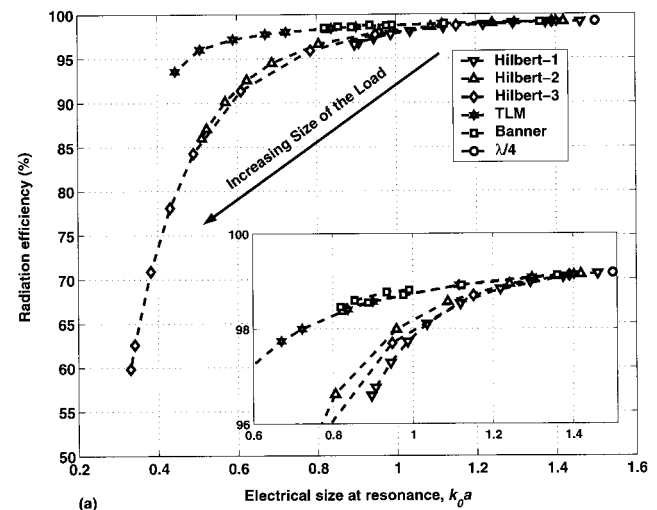


Figure 9 (a) Radiation efficiency and (b) quality factor at self-resonance of Hilbert curves as loads of monopoles compared with conventional top-loaded monopoles (loaded with a circular plate), banner monopoles and a $\lambda/4$ monopole. All of the monopoles have the same overall height

reached with, for example, top-loaded monopoles with circular plates, although the proposed geometry has the advantage that it can be easily manufactured with PCB photo-etching technology. However, even in this case, better results are observed using top-loaded MLMs. Nevertheless, it must be noticed that MLM antennas have been designed to fully occupy the available space in the given hemisphere that encloses the antenna.

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MULTIPOLE THEORY ANALYSIS OF THE CAPACITANCE MATRIX OF CIRCULAR ROD ARRAYS

Qinhong Zheng,¹ Hua Zeng,¹ Shaoquan Jiang,¹ Manyi Li,¹ Ming Song²

¹ School of Physics and Electronic Information
Yunnan Normal University
Kunming 650092, P. R. China

² Science Hall
Yunnan University
Kunming 650091, P. R. China

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ABSTRACT: The multipole theory (MT) method is exploited as a simple, accurate, and efficient tool for determining the capacitance matrix of circular rods located inside various grounded structures. The solution of the boundary-value problem associated with circular rod arrays is represented by the generalized MT formula, and the procedure to obtain the capacitance matrix of circular rod arrays is described. Representative computational examples, including the novel circular coaxial coupler, three circular rod arrays in a rectangular trough, and the cable consisting of nine circular inner conductors, are given to validate the theory, and to demonstrate the accuracy and flexibility of the MT method. © 2003 Wiley Periodicals, Inc. *Microwave Opt Technol Lett* 39: 49–52, 2003; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.11123

Key words: multipole theory; capacitance matrix; circular rod array

1. INTRODUCTION

Using circular rods in building microwave components (filters and couplers) is very popular because of its manufacturing advantage, as compared to rectangular bars. For these and other merits, a large number of realized components have been described, which indicate investigators' interest in the theoretical and experimental approach to the capacitance matrix of circular rods located inside various grounded structures [1–8].

The capacitance matrix (per unit length in the axial direction) of circular rod arrays can be determined by solving the two-dimensional (2D) Laplace's equation on the cross section of transmission lines. Recently, Zheng et al. [9] developed a new approach, the multipole theory (MT) method, for calculating 2D Laplace's equation boundary-value problem, and the MT method has been applied to the computation of the characteristic impedance of the coaxial line (containing one conductor) and the coupled transmission line (containing two conductors) [9–12]. In this paper, the MT method is applied to the analysis of capacitance matrix of circular rod arrays, that is, the multiconductor transmission line with complex cross section (containing more than two cylindrical conductors). The solution of the boundary-value problem related to circular rod arrays is represented by the generalized MT formula. To illustrate the application of the MT method for the capacitance matrix of a novel circular coaxial coupler, three circular rod arrays in a rectangular trough and the cable consisting of nine circular inner conductors are given to validate the theory.

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