On the Influence of Fractal Dimension on Radiation Efficiency and Quality Factor of Self-Resonant Prefractal Wire Monopoles

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Introduction (i)

- Fractal geometries demand attention in Antennas
  - Self-similarity properties
  - Fractionary Dimension
    - Multiband Antennas
    - Size-Reduced Antennas

- The measure of dimension of fractal curves does not correspond with their topological dimension
  - B. Mandelbrot
  - \( D > D_T \)

- A finite area/volume can enclose a fractal curve with infinite length.
Introduction (ii)

- In Antennas this ability could be used to pack wires in small spaces...

- First attempts using fractal curves as Antennas were
  - Minkowski loops
  - Koch monopoles

- Both designs showed an **expected** reduction of the resonant frequency as the fractal iteration of the monopole was increased.
As the number of iterations increased, the $Q$ was observed to approach the fundamental limit:

$$Q = \frac{1}{ka} + \frac{1}{(ka)^3}$$

$a$: radius of the smallest sphere that surrounds the antenna; 
$k$: wave number at the operating frequency.

R.C. Hansen: "To obtain performance closer to the minimum Q curve the spherical volume must be used more effectively; a dipole is essentially one dimensional".

The Koch curve is exploiting more efficiently its surrounding volume.

It was suggested that fractal curves, with fractal dimension $D > 1$ (linear dipole), could become more efficient small antennas.
This Work (i)

- Assessment of the relation (if any) among fractal dimension on the $Q$ factor and the $\eta$ of electrically small self-resonant prefractal wire monopoles. Comparison with Euclidean structures.
- Electrically small antenna: enclosed into a radiansphere ($a=\lambda/2\pi$) \( k_0a<1 \)
- Self-resonant monopoles: no need for external compensation of the reactive part of the impedance.
- The analysis is carried out through simulations and measurements using planar monopoles (easy fabrication procedures and baluns not needed).
This Work (ii)

- Fractals analyzed in the range: $1 < D \leq 2$
  - Koch monopoles: $D=1.26$
  - Sierpinski Arrowhead monopole: $D=1.58$
  - Hilbert monopole: $D=2.00$
  - Peano monopole: $D=2.00$

- Technological limitations: use of prefractals.

- Monopoles generated from an IFS algorithm.

\[
A_n = W[A_{n-1}]
\]
\[
W[A] = w_1[A] \cup w_2[A] \cup ... \cup w_N[A]
\]
This Work (iii)

- Performance comparison with Euclidean one-dimensional structures:
  - $\lambda/4$ monopole
  - *Intuitively* generated monopoles that fill $k_0a$: Meander line
    Loaded monopoles
This Work (iv)

Koch monopoles

Sierpinski Arrowhead monopoles

Peano monopoles

Hilbert monopoles

Meander line Loaded Monopoles
Simulations (i)

- Monopoles resonant at ~800 MHz.
- Copper wire, radius: 0.2 mm.
- Method of Moments software: NEC.
  - $0.2 \text{ GHz} \leq f \leq 1.2 \text{ GHz}$
  - $0.001 \leq \Delta/\lambda \leq 0.01$  \quad \text{2.5} \leq \Delta/b$
  - Extended Thin Wire Kernel

- Computation of
  - Radiation efficiency
    \[ \eta = \frac{R_r}{R_r + R_\Omega} \]
  - Quality factor
    \[ Q = \frac{\omega}{2R_r} \left( \frac{dX_{in}}{d\omega} + \frac{X_{in}}{\omega} \right) \]
Simulations (ii)

lossless $Q$
Measurements (i)

- Fabricated structures using standard PCB techniques.
- FR4 substrate, 0.25 mm thick, 35 µm etching, 0.35 mm strips width.
- 80 cm x 80 cm ground plane.
- SMA connector.
- Input impedance measurements using a VNA.
- Measurements inside an anechoic chamber.
Measurements (ii)

- Radiation efficiency and quality factor are measured with the Wheeler cap method.

@ at resonance $f_0$:

\[
Z_{\text{in}} = R_r + R_\Omega + jX_{\text{in}}
\]

@ RLC model:

\[
\eta = \frac{R_r}{R_r + R_\Omega} = \frac{\text{Re}\{Z_{\text{in}}\} - \text{Re}\{Z_{\text{cap}}\}}{\text{Re}\{Z_{\text{in}}\}}
\]

\[
Q = \frac{\omega}{2R_r} \left[ \frac{dX_{\text{in}}}{d\omega} + \frac{X_{\text{in}}}{\omega} \right]
\]
Measurements (iii)

- Cylindrical cap (height: 12.5 cm; diameter: 6 cm)

Modes inside a cylindrical cap excited by a \( \frac{\lambda}{4} \) monopole resonant at 694 MHz and skewed 10°.
Measurements (iv)

- Impedances are measured using
  - the electrical delay of the VNA to set the reference plane of the monopoles;
  - or by a rotation on the Smith chart to adjust the model of the antenna to an RLC circuit (McKinzie III method).
Measurements (iv)

Expected differences between measurements and simulations:

- wires/strips (~same electrical section)
- dielectric substrate freq. shift/additional losses
- real conductivity of copper
- soldering losses additional losses
- no connector/connector additional losses
- finite ground plane
- contact between cap and ground plane
- ...
- NEC limitations when segments are close together
Measurements

Prefractal monopoles

Koch monopoles

Peano monopoles

Sierpinski Arrowhead monopoles

Hilbert monopoles
Measurements (vi)

Euclidean monopoles

\(\lambda/4\) monopoles
Meander Line loaded Monopoles
Measurements (vii)

lossless $Q$
Conclusion

- First iterations of fractals of high $D$ seem interesting structures (small $k_0a$, low $Q$, high $\eta$) but...

- Other non-fractal geometries (ML monopoles filling the radianlength volume) achieve better performances with more degrees of freedom in their design.
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Quality factor comparison: SIMs vs MEASs

lossless $Q$

simulations

measurements
Rad. efficiency comparison: SIMs vs MEASs

simulations

measurements
# SIMd and MEASd Values

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<th>Antenna</th>
<th>D</th>
<th>$k_{0a}$ Sim’d</th>
<th>$Q$ Sim’d</th>
<th>$\gamma$ Sim’d</th>
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