Are Prefractal Monopoles Optimum Miniature Antennas?

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Introduction: Small Antennas (i)

- Maximum dimension less than the radian length (Wheeler): $ka < 1$
- Radiation pattern: doughnut-like.
- Directive gain: 1.5
- Radiation resistance:
  \[ R_{\text{rad small dipole}} = 80(ka)^2 \]
  \[ R_{\text{rad small loop}} = 20\pi^2(ka)^4 \]
- Limitation in bandwidth.

Image from: http://www.elliskaiser.com/doughnuts/tips.html
Introduction: \( Q \) (ii)

- Modelling the antenna as a resonant circuit \( Q \) could be used as a figure of merit:

\[
Q = \frac{2\omega W_e}{P_r} \quad W_e > W_m \quad Q = \frac{2\omega W_m}{P_r} \quad W_m > W_e
\]

- Fundamental limitation for linearly polarized antennas (propagation of only \( \text{TM}_{01} \) or \( \text{TE}_{01} \) spherical modes)^^:

\[
Q = \frac{1}{ka} + \frac{1}{(ka)^3}
\]

\( a \): radius of the smallest sphere enclosing the antenna; \( k \): wave number at the operating frequency.
Introduction: $Q$ (iii)

- Fractional bandwidth measured from the normalized spread between the half-power frequencies:

$$Q = \frac{1}{\text{Bandwidth}} = \frac{f_{\text{center}}}{f_{\text{upper}} - f_{\text{lower}}}$$

- $Q < 2$ : imprecise but useful (potentially broad band)
- $Q >> 1$ : good aprox. for Bandwidth$^{-1}$
  - narrow bandwidth
  - large frequency sensitivity
  - high reactive energy stored in the near zone of the antenna
  - large currents
  - high ohmic losses
Introduction: Objective (iv)

- Challenge: efficient radiation on large bandwidths with small antennas.
- Effective radiation associated with a proper use (TM$_{01}$ or TE$_{01}$) of the volume that encloses the antenna and a dipole is one-dimensional.
- Some prefractals have the ability to fill the space thanks to their $D > D_T$.
- Space-filling prefractals seem interesting structures to build size-reduced or small antennas, but...

... are they effective enough?
Introduction: Objective (v)

- Alternatives investigated:
  - Prefractal curves as antennas: performances in terms of $Q$ and $\eta$ of several monopole configurations studied.
    - Planar prefractals
    - 3D prefractals
  - Prefractal curves as top-loading of antennas.
  - GA design of fractals to achieve better performances.

- $Q$ and $\eta$ computed using the RLC model of the antenna at resonance

$$Q = \frac{\omega}{2R_r} \left( \frac{dX_{in}}{d\omega} + \frac{X_{in}}{\omega} \right), \quad \eta = \frac{R_r}{R_r + R_\Omega}$$
Planar Prefractals (i)

- Fractals are the attractors of infinite iterative algorithms: IFS (or NIFS).

\[
A_n = W[A_{n-1}]
\]

\[
W[A] = w_1[A] \cup w_2[A] \cup ... \cup w_N[A]
\]

\[w_i[A]: \text{ affine transformation scale - rotation - translation}\]

\[
\lim_{n \to \infty} A_n = \lim_{n \to \infty} W[A_{n-1}] = A_\infty
\]

We are limited to the use of prefractals.
Planar Prefractals (ii)

- Several electrically small planar self-resonant wire prefractal monopoles simulated and measured: $1 < D \leq 2$
- Comparison of performance with some Euclidean monopoles.
Planar Prefractals (iii)

- Though increasing complexity, quite similar behavior.
- Far away from the fundamental limit.
- Intuitively generated monopoles perform better.

measured results
Planar Prefractals (iv)

- Increasing ohmic losses with intricacy and iteration.
- Worst results than simulated due to the substrate.

measured results
3D Prefractals (i)

- 2D prefractals are far from the fundamental limit.
- 2D antennas do not use effectively the space inside the randiansphere \((k_0a<1)\).
- Do 3D antennas perform better because of their greater use of space?

- 3D-Hilbert monopoles are a continuous mapping of a segment into a cube.
- A monopole based on a 3D-Hilbert curve was simulated.
3D Prefractals (ii)

- In the first segments the current distribution tends to be more uniform.
- First segments do radiate and the rest act as a load.
- Non-radiating wire length with high currents: increase in ohmic losses.
3D Prefractals (iii)

- High reductions in size but unpractical values of $\eta$ and $Q$.

<table>
<thead>
<tr>
<th>Antenna</th>
<th>Total Wire Length (cm)</th>
<th>Electric Size at Resonance, $ka$</th>
<th>Resonant Frequency (MHz)</th>
<th>Radiation Resistance at Resonance (Ohms)</th>
<th>Input Resistance at Resonance (Ohms)</th>
<th>Efficiency (%)</th>
<th>Quality Factor</th>
<th>Fundamental Limit, $Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hilbert 3D-1</td>
<td>61.78</td>
<td>0.71</td>
<td>127,5</td>
<td>2.47</td>
<td>3.24</td>
<td>76.6</td>
<td>126</td>
<td>4</td>
</tr>
<tr>
<td>Hilbert 3D-2</td>
<td>184,99</td>
<td>0.25</td>
<td>51.4</td>
<td>0.16</td>
<td>1.67</td>
<td>9.4</td>
<td>2091</td>
<td>70</td>
</tr>
<tr>
<td>Hilbert 3D-3</td>
<td>642,61</td>
<td>0.10</td>
<td>21.0</td>
<td>0.01</td>
<td>3.07</td>
<td>0.5</td>
<td>31966</td>
<td>1088</td>
</tr>
</tbody>
</table>

@ copper wire $h=89.8$ mm $\neq 0.4$ mm

1\textsuperscript{st} iteration

$h=15$ mm $s=27$ mm

2\textsuperscript{nd} iteration

$h=5$ mm $s=17$ mm

3\textsuperscript{rd} iteration

$h=10$ mm $s=23$ mm
Prefractal Loading (i)

- While characterizing prefractal monopoles, we observed:
  - high $Q$ values $\rightarrow$ high stored energy
  - a strong dependence of $\eta$ and $Q$ with the length of the first segment of the prefractal

- Analysis of prefractals (Hilbert) as top loads for shorting monopoles.
- Comparison with a *banner* monopole.
- Comparison with a conventional top loaded monopole (circular plate).
Prefractal Loading (ii)

- Modelled antennas...

@ copper wire  $h=89.8$ mm  $\phi=0.4$ mm  $\Delta/a>2.5$  $\Delta/\lambda<0.01$
Prefractal Loading (iii)

- High $\eta$ and low $Q$ when small loads used.
- Electrically smaller self-resonant monopoles when increasing the relative size of the prefractal.
- $Q$ increases with the iteration of the prefractal for almost the same $\eta$, but the improvement is not significant.
- Pre-fractals admit greater size-reductions than conventional TLM, though unpractical $Q$ and $\eta$. 
GA Design (i)

- GA multiobjective optimization: design of wire Koch-like antennas optimized in terms of $Q$, $\eta$ and electrical size.

@ $h=6.22 \text{ cm} \ w=1.73 \text{ cm}$
GA Design (ii)

- Optimization on $Q-\eta-ka$: from the Pareto front 3 designs with the same wire length ($L \sim 10.22$ cm) have been selected and analyzed.

<table>
<thead>
<tr>
<th>Antenna Type</th>
<th>Resonant Frequency (MHz)</th>
<th>Quality Factor</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Koch</td>
<td>864.5</td>
<td>13.57</td>
<td>96.8</td>
</tr>
<tr>
<td>Meander</td>
<td>826.5</td>
<td>12.67</td>
<td>97.19</td>
</tr>
<tr>
<td>Zigzag</td>
<td>824</td>
<td>13.99</td>
<td>96.79</td>
</tr>
</tbody>
</table>

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</tr>
</thead>
<tbody>
<tr>
<td>Koch</td>
<td>905</td>
<td>12.67</td>
<td>87.64</td>
</tr>
<tr>
<td>Meander</td>
<td>850</td>
<td>12.60</td>
<td>88.78</td>
</tr>
<tr>
<td>Zigzag</td>
<td>870</td>
<td>13.89</td>
<td>87.34</td>
</tr>
</tbody>
</table>
GA Design (iii)

- GA multiobjective optimization: design of Euclidean planar structures with better performances than prefractals for the same electrical size.
GA Design (iv)

- Optimization on $Q$-$\eta$-$ka$ : 12-wires meander and zigzag antennas.

Pareto fronts
Conclusion (i)

- Small planar prefractal monopoles do not perform better than conventional Euclidean structures.
- Better $\eta$ and $Q$ factors when the (Hilbert) prefractal is used as a top-load than as an antenna but higher $ka$ ratios.
- 3D prefractal designs use more space but are unpractical designs as radiating elements.
- Even in the case of GA optimized prefractals Euclidean antennas achieve better performance than prefractals with less geometrical complexity.
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